A Layered Space-Time MIMO Detector With Parameterizable Performance

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Abstract—This brief presents an improved layered space-time symbol detection algorithm for multiple-input multiple-output (MIMO) wireless systems. The proposed detection scheme utilizes a different layer ordering strategy than the so-called optimal ordering used in the conventional Bell Laboratories Layered Space-Time (BLAST) algorithm. To calculate the nulling vectors and their associated ordering, instead of using direct numerical techniques, we utilize a numerically-stable iterative solution. It is shown that the performance of the proposed detector approaches closely that of an optimal maximum likelihood (ML) detector at the expense of greater computational complexity compared to BLAST. Utilizing our proposed parameterizable detector, one can trade off between the computational requirement and the error rate performance.

Index Terms—MIMO detection, layered space-time decoding, BLAST, generalized eigenvalue decomposition.

Notation: x^* denotes the complex conjugate of x, $|\mathbb{Q}|$ denotes the cardinality of signal constellation \mathbb{Q} , $\mathbb{E}\{\cdot\}$ denotes the expectation operator, $\|.\|$ denotes the Euclidean norm, I denotes the identity matrix, $(\cdot)^{-1}$ denotes the inverse of a matrix, $(\cdot)^*$ denotes the denotes the conjugate transpose (Hermitian) of a matrix.

I. INTRODUCTION

In a spatial multiplexing MIMO system with n_T transmitting antennas and n_R receiving antennas, the high-rate data stream is demultiplexed into n_T parallel sub-streams and the transmitter sends n_T independent symbols s_j simultaneously over a richly-scattered wireless channel, where $j = 1, \ldots, n_T$, chosen from a finite complex-valued signal constellation \mathbb{Q} . The n_R -dimensional vector of received signal y can be written as follows:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \sum_{j=1}^{n_T} \mathbf{h}_j s_j + \mathbf{n},$$
 (1)

where s denotes the $n_T \times 1$ transmitted signal vector and n is an $n_R \times 1$ noise vector with independent identically distributed (i.i.d.) circularly-symmetric complex Gaussian components with zero mean and unit variance, i.e., the components are from $\mathcal{CN}(0, 1)$. The $n_R \times n_T$ channel matrix is denoted by $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_{n_T}]$, where \mathbf{h}_k is the column vector of complex transfer gains from the k-th transmitter antenna to all n_R receiver antennas. For a flat-fading Rayleigh channel model with no line-of-sight, each element h_{ij} in \mathbf{H} (i.e., the gain from transmitter antenna j to receiver antenna i) is an i.i.d. circularly-symmetric complex Gaussian random variable from $\mathcal{CN}(0,1)$ [1].

The task of the MIMO detector is to estimate the symbol vector s from the received signal vector y. Maximum likelihood (ML) detection provides optimal error rate performance for uncoded MIMO systems. At the receiver, assuming that the channel matrix H is known (or estimated perfectly), an ML detector computes an estimate \hat{s} for each transmitted space-time (ST) symbol s using

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathbb{Q}^{n_T}}\left\{\left\|\mathbf{y} - \mathbf{H}\,\mathbf{s}\right\|\right\}$$
(2)

where the minimum is sought over all possible n_T -element ST symbols $s \in \mathbb{Q}^{n_T}$. Because an ML detector requires an exhaustive search over a typically large set of n_T possible transmitted ST symbols in \mathbb{Q}^{n_T} , its computational complexity grows exponentially with the number of transmit antennas and the signal constellation cardinality $|\mathbb{Q}|$. Simple linear detection schemes use low complexity filtering, designed using the zero-forcing (ZF) criterion or the minimum mean-square error (MMSE) criterion. While the ZF and MMSE detection schemes are computationally-efficient, their bit error rate performance is far from that of optimal ML detection [2].

Numerous detection algorithms have been proposed over the last decade with error rate performances and computational complexities residing between those of linear detection and optimal ML detection. In this brief we propose a layered detection algorithm similar to conventional Bell Laboratories Layered Space-Time (BLAST) with the following merits: (i) its error performance approaches closely that of a ML detector; (ii) it uses a numerically-stable iterative solution instead of a direct but computationally-intensive solution; (iii) its complexity is not variable, which makes the detection process predictable (deterministic); and (iv) it can be parameterized to readily trade off the computational requirement with the error rate performance. These merits are obtained with a computational complexity that is greater than that of the BLAST detection algorithm.

The rest of this brief is organized as follows. Section II briefly reviews MIMO detection schemes. Section III presents our proposed detection scheme. An iterative solution is utilized for an efficient implementation of the proposed technique. Section IV presents the simulation results. Section V presents

a variation of the proposed detector with parameterizable complexity. Finally, Section VI makes some concluding remarks.

II. BRIEF OVERVIEW OF MIMO DETECTION SCHEMES

Simple linear detection schemes use low complexity filtering to eliminate the multiple-stream interference (MSI). For example, ZF detection uses the Moore-Penrose pseudo inverse $\mathbf{G} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*$ of the channel matrix \mathbf{H} to reconstruct transmitted symbols as $\mathbf{s} = \mathbf{G}\mathbf{y} - \mathbf{G}\mathbf{n}$. Since the noise vector is weighted by the inverse singular values of \mathbf{H} , when the channel matrix becomes ill-conditioned in certain random-fading events, ZF can result in poor performance due to the noise enhancement. When the signal-to-noise ratio $(\text{SNR}) \sigma_s^2 / \sigma_n^2$ is estimated at the receiver, the MMSE criterion $\mathbf{G} = (\mathbf{H}^*\mathbf{H} + \frac{\sigma_s^2}{\sigma_s^2}\mathbf{I}_{n_T})^{-1}\mathbf{H}^*$ can be used instead to balance MSI mitigation with noise enhancement and thus minimize the total error. At low and mid-range SNR values, the MMSE outperforms the ZF receiver while at high SNR values the MMSE receiver converges to the ZF performance.

The non-linear vertical BLAST (V-BLAST) algorithm detects each layer (symbol) separately by using an iterative decision feedback approach, as shown in Algorithm 1. Since all n_T components of s utilize the same constellation Q, the weakest layer k, i.e., the layer with the smallest postdetection SNR $E\{|s_k|^2\}/(\sigma_n^2 ||\mathbf{g}_k||^2)$, where s_k denotes the transmitted signal from the k-th transmitter antenna and \mathbf{g}_k is the k-th row of G, will dominate the error performance of the system. Thus it was recommended in [3] that the detection algorithm start with the layer with the strongest post-detection SNR (i.e., corresponding to the row \mathbf{g}_k of \mathbf{G} with the minimum norm) and then proceed successively to detect the symbol of the layer with the next weakest SNR. As shown in Algorithm 1, after estimating and canceling s_k , h_k is zeroed and hence G must use a deflated version $H_{\bar{k}}$ of H in the next iteration, where $\mathbf{H}_{\bar{k}}$ denotes the matrix obtained by zeroing column k of **H**. The notation O(i) denotes the layer k, where $k \in \{1, \ldots, n_T\}$, that is to be detected at step *i*. Note that under the assumption of quasi-stationary block-fading channels, the channel variation is negligible over a coherence period and it changes independently from one period to another. Therefore, nulling vectors need be computed only once for every block of received symbols. For each layer k, first an interference nulling step tries to reduce the amount of interference towards s_k by multiplying the received signal y by a nulling vector \mathbf{g}_k . Second, symbol s_k is detected using the slicer function $\mathbb{Q}(\cdot)$, which returns the nearest symbol in the signal constellation Q to the estimated symbol \hat{s}_k . Finally, the predicted interference on the $n_T - 1$ other signals due to s_k can be subtracted from the received signal. V-BLAST proceeds iteratively through the above three steps until all n_T transmitted symbols are recovered.

Various non-exhaustive tree-based search detectors with optimal (or near-optimal) performance have been proposed. The main idea is to prune the exhaustive search space and thereby provide substantial computational saving over the ML bruteforce searching method. Depending on how the tree traversal

Algorithm 1 The MMSE V-BLAST algorithm.

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\hat{\mathbf{H}} = \mathbf{H}:
for (i = 1; i \le n_T; i++) do
    \mathbf{G} = (\hat{\mathbf{H}}^* \hat{\mathbf{H}} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I})^{-1} \hat{\mathbf{H}}^*;
    O(i) = k = \min_{j} \|\mathbf{g}_{j}\|^{2}; \{\text{Ordering}\}
    \mathbf{g}_i = \mathbf{G}(k, :);
    \hat{\mathbf{H}} = \hat{\mathbf{H}}_{\bar{k}};
end for
for (every received symbol vector y in a block) do
    for (i = 1; i < n_T) do
        k = O(i);
                                     {Nulling}
         \hat{s}_k = \mathbf{g}_k \mathbf{y};
                                     {Slicing}
         s_k = \mathbb{Q}(\hat{s}_k);
        if (i < n_T) then
                 \mathbf{y} = \mathbf{y} - \mathbf{h}_k s_k; {Cancelation}
         end if
    end for
    return s;
end for
```

algorithms carry out the non-exhaustive search problem [4], they generally fall into two main categories, namely the depthfirst search and breadth-first search methods. Sphere decoding (SD) is probably the most attractive depth-first approach [5], [6]. The performance of SD closely approaches that of ML with significantly reduced average complexity compared to ML detection. The SD algorithm reduces the number of candidate ST symbols to be considered in the tree-based search by finding the set $\{s\}$ of ST symbols closest to the received vector symbol y for which Hs lies within a hypersphere with given radius d_o , i.e., $\|\mathbf{y} - \mathbf{Hs}\|^2 < d_o^2$. Usually, d_o is adjusted according to the noise variance [6], [7]. Unfortunately, the size of the searched solution space, and hence its instantaneous throughput, is variable and is directly related to the channel conditions and also the operating SNR [8]. When the channel is ill-conditioned (i.e., a channel whose condition number is much greater than one) [9], the computational complexity of SD substantially increases and approaches that of ML detection (i.e., exponential in n_T) [5], [10]. However, for sufficiently large SNR it behaves polynomially in n_T as long as n_T is not too large [6]. In addition to the average computational complexity (i.e., averaged over a sufficient number of channel realizations) of SD, which approaches that of ML detection [11], the variable complexity of SD makes this scheme inconvenient for practical implementations where data needs to be processed at a constant rate. To overcome the limitations of SD, the fixed-sphere decoding (FSD) searches over only a fixed number of lattice vectors Hs, generated by a subset of all constellation points around the received vector y, independent of the channel conditions and noise level, yielding a fixed complexity algorithm [12]. Moreover, FSD does not suffer from the sequential nature of the SD and all the paths in the tree structure can be searched in parallel. Nevertheless, the complexity of the FSD is high [13] and its performance falls short of being optimum [14]. Similarly, the K-Best breadthfirst search algorithm [15] provides a deterministic constant throughput which is independent of the channel realization and the SNR; however, *K*-Best search does not necessarily yield ML performance.

III. PROPOSED LAYERED SPACE-TIME MIMO DETECTION TECHNIQUE

The non-linear BLAST detection algorithm remains an attractive solution due to its substantial lower computational complexity compared to ML and improved performance relative to the linear MMSE algorithm; however, there still remains a substantial gap between the performance of V-BLAST [16] and that of optimal ML detection [3]. Consider a spatial multiplexing MIMO system that experiences a richly-scattered wireless channel. We are interested in finding nulling vectors and their associated ordering that maximizes the signal-to-noise-plus-interference ratio (SNIR). Let g_k denote the nulling vector associated with the *k*-th transmitted symbol s_k . If we multiply both sides of (1) by g_k^* , then we can write (1) as

$$\mathbf{g}_{k}^{*}\mathbf{y} = \mathbf{g}_{k}^{*}\mathbf{h}_{k}s_{k} + \sum_{j \neq k} \mathbf{g}_{k}^{*}\mathbf{h}_{j}s_{j} + \mathbf{g}_{k}^{*}\mathbf{n},$$
(3)

where the first term on the right hand side in (3) denotes the signal, the second term denotes the interference and the last term denotes the noise. Then the SNIR γ_k , which is a non-negative real-valued number corresponding to \mathbf{g}_k , can be written as follows:

$$\gamma_{k} = \frac{\mathrm{E}_{s}\left\{|s_{k}|^{2}\mathbf{g}_{k}^{*}\mathbf{h}_{k}\mathbf{h}_{k}^{*}\mathbf{g}_{k}\right\}}{\mathrm{E}_{s}\left\{\sum_{j\neq k}|s_{j}|^{2}\mathbf{g}_{k}^{*}\mathbf{h}_{j}\mathbf{h}_{j}^{*}\mathbf{g}_{k}\right\} + \mathrm{E}_{n}\left\{\mathbf{g}_{k}^{*}\mathbf{nn}^{*}\mathbf{g}_{k}\right\}}$$
$$= \frac{\sigma_{s}^{2}\mathbf{g}_{k}^{*}\mathbf{h}_{k}\mathbf{h}_{k}^{*}\mathbf{g}_{k}}{\sigma_{s}^{2}\sum_{j\neq k}\mathbf{g}_{k}^{*}\mathbf{h}_{j}\mathbf{h}_{j}^{*}\mathbf{g}_{k} + \mathbf{g}_{k}^{*}\sigma_{n}^{2}\mathbf{I}_{n_{R}}\mathbf{g}_{k}}$$
$$= \frac{\mathbf{g}_{k}^{*}\left[\mathbf{h}_{k}\mathbf{h}_{k}^{*}\right]\mathbf{g}_{k}}{\mathbf{g}_{k}^{*}\left[\mathbf{H}_{\bar{k}}\mathbf{H}_{\bar{k}}^{*} + \frac{\sigma_{s}^{2}}{\sigma_{s}^{2}}\mathbf{I}_{n_{R}}\right]\mathbf{g}_{k}}$$
(4)

For a clearer representation, we define two matrices

$$\mathbf{A}_{k} = \mathbf{h}_{k}\mathbf{h}_{k}^{*} \tag{5}$$

$$\mathbf{B}_{k} = \mathbf{H}_{\bar{k}}\mathbf{H}_{\bar{k}}^{*} + \frac{\sigma_{n}^{2}}{\sigma_{s}^{2}}\mathbf{I}_{n_{R}}$$
(6)

Note that \mathbf{A}_k is a positive-semidefinite (i.e., $\mathbf{g}_k^* \mathbf{h}_k \mathbf{h}_k^* \mathbf{g}_k \ge 0$) Hermitian matrix of rank one and \mathbf{B}_k is a positive-definite matrix (i.e., $\mathbf{g}_k^* \left[\mathbf{H}_{\bar{k}} \mathbf{H}_{\bar{k}}^* + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I} \right] \mathbf{g}_k > 0$) [9]. The SNIR γ_k can now be written as:

$$\gamma_k = f(\mathbf{g}_k = \mathbf{x}_k) = \frac{\mathbf{x}_k^* \mathbf{A}_k \mathbf{x}_k}{\mathbf{x}_k^* \mathbf{B}_k \mathbf{x}_k}$$
(7)

One approach to finding the maximum $f(\mathbf{x}_k)$ (i.e., the maximum SNIR) is to first obtain its gradient

$$\nabla f(\mathbf{x}_k) = \frac{2\mathbf{x}_k^* \mathbf{A}_k(\mathbf{x}_k^* \mathbf{B}_k \mathbf{x}_k) - 2\mathbf{x}_k^* \mathbf{B}_k(\mathbf{x}_k^* \mathbf{A}_k \mathbf{x}_k)}{(\mathbf{x}_k^* \mathbf{B}_k \mathbf{x}_k)^2}$$
$$= \frac{2(\mathbf{x}_k^* \mathbf{A}_k - \mathbf{x}_k^* \mathbf{B}_k \gamma_k)}{\mathbf{x}_k^* \mathbf{B}_k \mathbf{x}_k}$$
$$= \frac{2(\mathbf{A}_k \mathbf{x}_k - \gamma_k \mathbf{B}_k \mathbf{x}_k)}{\mathbf{x}_k^* \mathbf{B}_k \mathbf{x}_k}$$
(8)

where we use the Hermitian properties and the definition of the partial derivative $\partial(\mathbf{x}^* \mathbf{A} \mathbf{x}) / \partial \mathbf{x} = 2\mathbf{x}^* \mathbf{A}$ [9]. By setting the numerator of (8) equal to zero, we obtain

$$\mathbf{A}_k \mathbf{x}_k = \gamma_k \mathbf{B}_k \mathbf{x}_k \tag{9}$$

Equation (9) defines a generalized eigenvalue problem, where $\mathbf{x}_k \neq \mathbf{0}$ is the generalized eigenvector associated with the generalized eigenvalue γ_k of the matrix pair $(\mathbf{A}_k, \mathbf{B}_k)$ [9]. A generalized eigenvalue decomposition technique can be used to find the eigenpairs (γ_k, \mathbf{g}_k) [9].

$$(\gamma_k, \mathbf{g}_k) = \operatorname{geig}\left(\mathbf{h}_k \mathbf{h}_k^*, \ \mathbf{H}_{\bar{k}} \mathbf{H}_{\bar{k}}^* + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_{n_R}\right)$$
(10)

The process of finding nulling vectors using the generalized eigenvalue decomposition in (10) must be repeated for the remaining $n_T - 1$ layers.

Following the so-called optimum ordering [3] proposed in V-BLAST, the k-th nulling vector \mathbf{g}_k is computed as the generalized eigenvector associated with the k-th largest eigenvalue γ_k . In [17] we proposed to use a different ordering strategy. Our ordering scheme was motivated by the observation that the performance of the V-BLAST detector is especially limited by the worst sub-channel. Therefore, accurate detection of the weakest layer has a significant impact on the error rate performance of the system. We start with the worst sub-channel and detect the weakest layer optimally using an exhaustive search over all possible transmitted symbols from constellation \mathbb{Q} . The remaining $n_T - 1$ layers then use the conventional V-BLAST "best-first" ordering.

The pseudo-code of the proposed detection scheme is shown in Algorithm 2. After determining the ordering and calculating the nulling vectors using generalized eigenvalue decompositions, the detection process starts out by canceling the contribution of a tentative candidate symbol $s_k^j \in \mathbb{Q}$ from the weakest layer k of the received signal y, where $j = 1, \ldots, |\mathbb{Q}|$ and $|\mathbb{Q}|$ is the cardinality of signal constellation. After detecting the remaining $n_T - 1$ layers using the conventional sequence of ordered nulling, slicing and cancelation, an error metric $\xi_j = \|\mathbf{Hs}_j - \mathbf{y}\|^2$ associated with the tentative symbol s_k^j is computed, where $\mathbf{s}_j = [s_1^j, s_2^j, \cdots, s_{n_T}^j]^T$ is the detected symbol vector, assuming that symbol s_k^j was transmitted from antenna k. This process is then repeated $|\mathbb{Q}| - 1$ times for all the remaining tentative candidate symbols in the constellation \mathbb{Q} . Then the algorithm chooses the symbol vector \mathbf{s}_i with the smallest error metric ξ_j as the detected symbol vector. We will refer to the detection process of symbol vector s_j for each tentative symbol s_k^j as a sub-detector.

IV. SIMULATION RESULTS

The algorithm proposed in [17] used the *direct* matrix inversion techniques [9], [18] to calculate the nulling vectors and their associated ordering that maximizes the *signal-to-noise* ratio using the MMSE criterion $\mathbf{G} = (\mathbf{H}^*\mathbf{H} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_{n_T})^{-1}\mathbf{H}^*$. In this work we instead use nulling vectors and an ordering that maximize the *signal-to-noise-plus-interference ratio* using the generalized eigenvalue decomposition technique. Hence, n_T

Algorithm 2 Proposed detection algorithm

 $\hat{\mathbf{H}} = \mathbf{H}$: for $(i = 1; i \le n_T; i + +)$ do if i > 1 then $(\gamma, \mathbf{g}_i) = \max \operatorname{geig}\left(\hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^*, \ \hat{\mathbf{H}} \hat{\mathbf{H}}^* + \frac{\sigma_n^2}{\sigma^2} \mathbf{I}_{n_R}\right);$ else $(\gamma, \mathbf{g}_i) = \min_i \operatorname{geig}\left(\hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^*, \ \hat{\mathbf{H}}\hat{\mathbf{H}}^* + \frac{\sigma_n^2}{\sigma_z^2} \mathbf{I}_{n_R}\right);$ end if $O(i) = \gamma;$ $\hat{\mathbf{H}} = \hat{\mathbf{H}}_{\bar{\gamma}};$ end for for (every received symbol vector y in a block) do for $(j = 1; j \le |Q|; j + +)$ do $\hat{\mathbf{y}} = \mathbf{y};$ k = O(1); {Get weakest layer} $\hat{\mathbf{y}} = \hat{\mathbf{y}} - \mathbf{h}_k s_k^j$; {Cancel tentative symbol} for $(i = 2; i \leq n_T; i + +)$ do k = O(i); {Get strongest layer to be detected} $\hat{s}_k = \mathbf{g}_k \hat{\mathbf{y}}; \{\text{Nulling}\}$ $s_k = \mathcal{Q}(\hat{s}_k); \{ \text{Slicing} \}$ $s_k^j = s_k;$ if $(i < n_T)$ then $\hat{\mathbf{y}} = \hat{\mathbf{y}} - \mathbf{h}_k s_k; \{\text{Cancelation}\}$ end if end for $\xi_j = \|\mathbf{Hs}_j - \mathbf{y}\|^2;$ end for $\ell = \min(\xi_j);$ return \mathbf{s}_{ℓ} ; end for

matrix inversion operations in the algorithm proposed in [17] are replaced with n_T generalized eigenvalue decompositions.

Since \mathbf{B}_k is full rank and non-singular, the generalized eigenvalue problem $\mathbf{A}_k \mathbf{x}_k = \gamma_k \mathbf{B}_k \mathbf{x}_k$ can be solved by reducing it to a standard eigenvalue problem $\mathbf{B}_k^{-1} \mathbf{A}_k \mathbf{x}_k = \gamma_k \mathbf{x}_k$ or using techniques such as QZ decomposition [9]. Fig. 1 shows the SER of alternative detection schemes for a 4×4 uncoded MIMO system utilizing 16-QAM modulation over a Rayleigh fading channel. Simulation results show that the proposed detection scheme can achieve SER performance that essentially matches that of the ML detector.

It is shown in [19] that if \mathbf{B}_k is ill-conditioned with respect to inversion, the condition number is large and the direct methods may be numerically unstable and/or have large errors in computed eigenvalues and eigenvectors. An alternative approach to directly computing the eigenpairs (γ_k, \mathbf{g}_k) is to use *iterative* algorithms that converge almost always to a solution asymptotically within acceptable precision after a small number of iterations. We propose to calculate both the nulling vectors and their associated ordering using an iterative method. We are especially interested in a numerical algorithm that converges to the solution in a small number of iterations (e.g., three to five iterations). The Conjugate Gradient (CG) algorithm [20] is an iterative approach for the numerical solution of a system of linear equations whose matrix is symmetric (i.e., $\mathbf{Q} = \mathbf{Q}^*$) and positive definite (i.e., for all non-zero complex-valued vectors \mathbf{z} , $\mathbf{z}^*\mathbf{Q}\mathbf{z} > 0$). The



Fig. 1. Symbol error rate of ML and our proposed detection schemes for a 4×4 16-QAM MIMO system over a Rayleigh fading channel.

important property of the CG algorithm is that it takes at most n iterations to find a minimum of an n-dimensional quadratic function $f(\mathbf{x})$. An important point to note is that all eigenvalues of \mathbf{B}_k are positive and since \mathbf{B}_k is a Hermitian matrix, $\mathbf{x}_k^* \mathbf{B}_k \mathbf{x}_k$ is always real-valued. Thus the denominator of (8) is a positive scalar that does not change the direction in the convergence of $f(\mathbf{x})$ and, hence, does not play an important role in finding the direction of maximum rate of decrease for $f(\mathbf{x})$ in the CG algorithm. Moreover, the numerator of (8) can be written as follows:

$$\mathbf{A}_{k}\mathbf{x}_{k} - \gamma \mathbf{B}_{k}\mathbf{x}_{k} = \mathbf{A}_{k}\mathbf{x}_{k} - \frac{\mathbf{B}_{k}\mathbf{x}_{k}\mathbf{x}_{k}^{*}\mathbf{A}_{k}\mathbf{x}_{k}}{\mathbf{x}_{k}^{*}\mathbf{B}_{k}\mathbf{x}_{k}}$$
$$= \frac{\left((\mathbf{x}_{k}^{*}\mathbf{B}_{k}\mathbf{x}_{k})\mathbf{I} - \mathbf{B}_{k}\mathbf{x}_{k}\mathbf{x}_{k}^{*}\right)\mathbf{A}_{k}\mathbf{x}_{k}}{\mathbf{x}_{k}^{*}\mathbf{B}_{k}\mathbf{x}_{k}}$$
(11)

where, following the CG algorithm, we use the numerator of (11) as an approximation to the gradient vector.

Fig. 2 confirms that the SER performance of the proposed detection scheme, using the CG iterative algorithm to solve the generalized eigenvalue decompositions, closely matches that of ML detector. To ensure sufficient accuracy and a deterministic computation, we limit the number of iterations to four as a stopping criteria for computing each eigenpair when using the iterative CG algorithm. As can be seen in Fig. 2, both algorithms in [17], which used direct matrix inversion to calculate the nulling vectors and the associated ordering that maximizes the signal-to-noise ratio, and the proposed iterative algorithm, which uses the generalized eigenvalue decomposition technique to find the nulling vectors and the associated ordering that maximizes the signal-to-noise-plusinterference ratio, result in overlapping SER vs. SNR performance. An important point to note is that these simulations results are obtained using floating-point implementations of these algorithms. In practice, due to the higher hardware cost and complexity of floating-point hardware, fixed-point arithmetic is often preferred in very-large-scale integration (VLSI)

and field-programmable gate array (FPGA) implementations. Therefore, the choice of an algorithm depends on various factors, such as the computational complexity, numerical stability of its operations, round-off errors and desired accuracy, and required hardware resources and expected throughput. For example, iterative algorithms may have smaller storage requirements than direct methods [18] and become preferable for relatively large matrices or when the problem is close to singular [21].



Fig. 2. Symbol error rate of ML detection and our proposed detection using conjugate gradient algorithm with four iterations and the detector from [17] using matrix inversion.

V. PERFORMANCE-COMPLEXITY TRADE-OFF

An important property of Algorithm 2 is that the detection process for the transmitted symbol vector s corresponding to each possible choice of the tentative signal s^{j} (for the worst layer) can be performed independently and simultaneously, which leads naturally to a parallel implementation of the sub-detectors. For a compact implementation, one can implement only one instance of the sub-detector and time multiplex it among other $|\mathbb{Q}| - 1$ sub-detectors at the expense of lowering the symbol detection throughput. Note that as each sub-detector requires one fewer nulling and slicing operations (for the weakest layer), the computational complexity of the proposed detector is roughly $|\mathbb{Q}|$ times that of BLAST detection. This can be significant, especially for high-order modulation schemes. We propose to reduce the required computation of the proposed detector substantially by searching within a subset $\mathbb{Q}^* \subset \mathbb{Q}$ of constellation points within the weakest layer instead of an exhaustive search over the entire constellation points in Q. This subset should be chosen to minimize the computations, while also large enough to provide acceptable performance. This approach reduces the computational requirements compared to ML exhaustive search for the weakest layer considerably. Moreover, rather than customizing a newly-sized search circular search space with a variable radius around the weakest layer of each received vector y and dynamically calculating the constellation points within the circle, we propose to create a circle with the same radius d_o . Utilizing this approach, while reducing the search computations, one can readily trade off between the error rate performance and the required computation of the proposed detector. By choosing an appropriate radius d_o one can optimize the trade-off between the required amount of computational processing and error rate performance of the algorithm.

To perform the non-exhaustive search for the weakest layer, first an initial circle $S_o(d_o)$ with radius d_o is constructed around the origin. All possible constellation points within $S_o(d_o)$ can be precomputed and stored during the initialization step. The initial circle $S_o(d_o)$ is independent of the received signals and the channel conditions. The number n_S of points within $S_o(d_o)$ depends on the initial radius d_o and the signal constellation \mathbb{Q} . Every received signal vector y is decoded to **p** using the V-BLAST algorithm. A variety of algorithms can be used for preliminary decoding since the more accurate the initial decoding, the better the overall performance of the detection algorithm. Next, a displaced search circle $S(s_k, d_o)$ is constructed with radius d_{o} centered about the weakest symbol s_k of the preliminary decoded point **p** and the constellation points that lie outside $\mathcal{S}(s_k, d_o)$ are excluded from the search. For example, as shown in Fig. 4, 7 + 7j is the the symbol associated with the weakest layer in the preliminary decoded point **p** where $d_o = 3.2$. Instead of searching over all other 63 constellation points, as proposed in Algorithm 2, only the $n_s = 3$ constellation points $\{5 + 5j, 5 + 7j, 7 + 5j\}$ in the circle S(7 + 7j, 3.2) are considered.



Fig. 3. An example of the search circle around the weakest layer of decoded point in a 64-QAM modulation with $d_o = 3.2$.

Note that fixed-sphere decoding searches over lattice vectors **H**s generated by a subset of all constellation points around the received vector **y** for which **H**s lies within a *hypersphere* with a fixed radius d_o , i.e., $\|\mathbf{y} - \mathbf{Hs}\|^2 < d_o^2$. In the proposed algorithm we used a search *circle* with a fixed radius d_o around the weakest layer of each received vector **y** only and detect the weakest layer near-optimally using an exhaustive search within a subset $\mathbb{Q}^* \subset \mathbb{Q}$ of constellation points. The remaining $n_T - 1$

layers are detected using successive interference cancelation. Fig. 4 shows the symbol error rate of the reduced approach for a 4×4 64-QAM MIMO system with the radius of the search space set to $d_o = 6$ and $d_o = 3.2$. As shown, with increasing d_o the error rate decreases at the expense of increasing the computational complexity of the search within the circle.



Fig. 4. Symbol error rate of the reduced approach for 64-QAM and $d_o = 6$.

VI. CONCLUSIONS

Multiple antenna communication systems can achieve remarkably high data rates. We presented a layered detection scheme, similar to the conventional Bell Laboratories Layered Space-Time (BLAST) algorithm but with a different layer ordering. The proposed layered detector has higher complexity than the conventional BLAST detector, but it offers significantly improved performance that closely matches that of a maximum likelihood (ML) detector. In order to decrease the computational requirements, we proposed a conjugate gradient iterative algorithm to replace the direct numerical solution for generalized eigenvalue decompositions. Our simulation results show that using only three or four iterations, the performance of our proposed scheme closely matches that of optimal ML detection. One can also trade off computational complexity with the error rate performance using our proposed reducedcomplexity algorithm.

APPENDIX

A. CONJUGATE GRADIENT METHOD

The pseudo-code of the CG algorithm is shown in Algorithm 3. The search for a minimum of $f(\mathbf{x})$ starts at an arbitrary point \mathbf{x}_0 and traverses along a first direction vector \mathbf{d}_0 and stops at the point $\mathbf{x}_1 = \mathbf{x}_0 + \alpha_0 \mathbf{d}_0$, where $\alpha_0 > 0$ is the line search parameter that determines the length of the first step that minimizes $f(\mathbf{x})$ along the line. The direction with the maximum rate of increase for $f(\mathbf{x})$ at \mathbf{x}_i can be represented by the gradient vector $\nabla f(\mathbf{x}_i)$. Since $f(\mathbf{x})$ decreases most rapidly along the direction of a negative gradient, CG performs a line search for the first step along $\mathbf{d}_i = -\nabla f(\mathbf{x}_i)$. For the remaining steps, the CG algorithm uses a set of mutually conjugate directions in which the next search direction d_{i+1} is constrained to be conjugate (or Q-orthogonal) to the previous. Two vectors d_{i+1} and d_i are Q-orthogonal with respect to any symmetric positive definite matrix **Q** if $\mathbf{d}_{i+1}^* \mathbf{Q} \mathbf{d}_i = \mathbf{0}$. Thus, in the remaining steps a plane rather than a line, which is used in the steepest descent algorithm [9], is searched and it is guaranteed to produce a new, linearly-independent search direction. One vector will be chosen to be the normalized gradient vector \mathbf{g}_i and the other vector will be chosen to be $\mathbf{d}_{i+1} = \beta_i \mathbf{d}_i - \mathbf{g}_{i+1}$, where the coefficient β_i is given by $\mathbf{g}_{i+1}^*\mathbf{g}_{i+1}/\mathbf{g}_i^*\mathbf{g}_i$. The step length of α_i should be chosen so that d_i and d_{i+1} are conjugate. The iterative process continues until the minimum in $f(\mathbf{x})$ has been determined within a chosen accuracy ε .

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i = 0;	
$\mathbf{x}_i = \mathbf{h}_k / \mathbf{h}_k ;$	
$\mathbf{g}_i = \frac{\nabla f(\mathbf{x}_i)}{\ \nabla f(\mathbf{x}_i)\ _2};$	
$\mathbf{d}_i = -\mathbf{g}_i; \mathbf{d}_i = -\mathbf{g}_i;$	
$\eta_i = \mathbf{g}_i^* \mathbf{g}_i;$	
$\eta_{i+1} = \eta_i;$	
while $(i < i_{max} \text{ and } \eta_{i+1} > \varepsilon^2 \eta_0)$ do	
$\alpha_i = \frac{\eta_i}{\mathbf{d}_i^* \mathbf{B} \mathbf{d}_{i+1}};$	
$\mathbf{x}_{i+1} = \mathbf{x}_i + \alpha_i \mathbf{s}_i;$	
$\mathbf{g}_{i+1} = -\frac{\nabla f(\mathbf{x}_{i+1})}{\ \nabla f(\mathbf{x}_{i+1})\ _2};$	
$\eta_i = \eta_{i+1};$	
$\eta_{i+1} = \mathbf{g}_{i+1}^* \mathbf{g}_{i+1};$	
$\beta_i = \eta_{i+1}/\eta_i;$	
$\mathbf{d}_{i+1} = \beta_i \mathbf{d}_i + \mathbf{g}_{i+1};$	
i = i + 1;	
end while	

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